Problems Based on Biot Savart's Law

Objectives

After going through this lesson, the learners will be able to :

- Apply Biot Savart's law to
- Calculate the magnitude of magnetic field at a distance away from a current carrying conductor
- Predict the direction of magnetic field at a location using right hand rule
- Determine the magnetic field due to a straight conductor of finite size.
- Determine the magnetic field on the axis of a circular current loop.
- Solve problems using Biot Savart's law

Content Outline

- Unit syllabus
- Module wise distribution of unit syllabus
- Words you must know
- Introduction
- Application of Biot-Savart law
    - Magnetic field due to a straight conductor of finite size
    - Magnetic field on the axis of a circular loop
- Examples based on applications of Biot-Savart law
- Problems for practice
- Summary

Unit Syllabus

Magnetic Effects of Current and Magnetism-10 Modules

Chapter-4: Moving Charges and Magnetism

Concept of magnetic field, Oersted's experiment. Biot - Savart law and its application to the current carrying circular loop. Ampere's law and its applications to infinitely long straight wire. Straight and toroidal solenoids, Force on a moving charge in uniform magnetic and electric fields. Cyclotron. Force on a current-carrying conductor in a uniform magnetic field. Force between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in uniform magnetic field; moving coil galvanometer-its current sensitivity and conversion to ammeter and voltmeter.

Chapter-5: Magnetism and Matter

Current loop as a magnetic dipole and its magnetic dipole moment. Magnetic dipole moment of a revolving electron. Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis. Torque on a magnetic dipole (bar magnet) in a uniform magnetic field; bar magnet as an equivalent solenoid, magnetic field lines; Earth's magnetic field and magnetic elements.

Para, dia and ferro-magnetic substances, with examples; Electromagnets and factors affecting their strengths. Permanent magnets.

Module Wise Distribution-10 Modules

The above unit is divided into 10 modules for better understanding.

Module 1	• Introducing moving charges and magnetism • Direction of magnetic field produced by a moving charge • Concept of Magnetic field • Oersted's Experiment • Strength of the magnetic field at a point due to current carrying conductor • Biot-Savart Law • Comparison of coulomb's law and Biot Savart's law
Module 2	• Applications of Biot- Savart Law to current carrying circular loop, straight wire • Magnetic field due to a straight conductor of finite size • Examples
Module 3	• Ampere's Law and its proof • Application of ampere's circuital law: straight wire, straight and toroidal solenoids. • Force on a moving charge in a magnetic field • Unit of magnetic field • Examples
Module 4	• Force on moving charges in uniform magnetic field and uniform electric field.

	● Lorentz force ● Cyclotron
Module 5	● Force on a current carrying conductor in uniform magnetic field ● Force between two parallel current carrying conductors ● Definition of ampere
Module 6	● Torque experienced by a current rectangular loop in uniform magnetic field ● Direction of torque acting on current carrying rectangular loop in uniform magnetic field ● Orientation of a rectangular current carrying loop in a uniform magnetic field for maximum and minimum potential energy
Module 7	● Moving coil Galvanometer- ● Need for radial pole pieces to create a uniform magnetic field ● Establish a relation between deflection in the galvanometer and the current ● its current sensitivity ● Voltage sensitivity ● conversion to ammeter and voltmeter ● Examples
Module 8	● Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis. ● Torque on a magnetic dipole in a uniform magnetic field. ● Explanation of magnetic property of materials
Module 9	● Dia, Para and ferromagnetic substances with examples. Electromagnets and factors affecting their strengths, permanent magnets.
Module 10	● Earth's magnetic field and magnetic elements.

Module 2

Words You Must Know

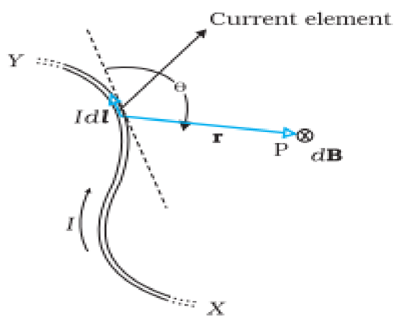
- **Coulomb's law:** The force of attraction or repulsion between two point charges is directly proportional to the product of two charges (q_1 and q_2) and inversely proportional to the square of the distance between them. It acts along the line joining them.
- **Electric current:** The rate of flow of charge with time.
- **Magnetic field lines:** It is a curve, the tangent to which a point gives the direction of the magnetic field at that point.
- **Maxwell's cork screw rule or right hand screw rule:** It states that if the forward motion of an imaginary right handed screw is in the direction of the current through a linear conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.
- **Biot-Savart law:** According to Biot-Savart law, the magnetic field *dB* at *P* due to the current element *idl* is given by

$$dB = \mu_0 \, Idl \sin \theta / 4\pi r^2$$

- **Right hand thumb rule or curl rule:** If a current carrying conductor is imagined to be held in the right hand such that the thumb points in the direction of the current, then the tips of the fingers encircling the conductor will give the direction of the magnetic lines of force.

Introduction

What is the Biot-Savart Law?



You will recall from our previous modules that electric fields and magnetic fields might seem different, but they're actually part of one larger force called the electromagnetic force. Charges that aren't moving produce electric fields. But when those charges do move, they instead create magnetic fields. Charges moving in an electric wire also produce magnetic fields. If we move a compass near to an electric wire, the compass needle changes direction or deflects.

The **Biot-Savart Law is a mathematical expression that describes the magnetic field created by a current-carrying wire, and allows you to calculate its strength at various points.**

To derive this law, we first take this equation for the electric field. This is the full version, where we use $\mu_0 / 4\pi$ instead of the electrostatic constant *k*. Since we're looking at a wire, we replace the charge *q* with *I dl*, which is the current in the wire multiplied by a length element in the wire. Basically it's treating this little chunk of the wire as our charge. And we also replace the electric field *E* with a magnetic field element *dB* because a moving charge produces a magnetic field, not an electric field.

Last of all, we have to realize that a current has a direction (unlike a charge). So we need to make sure the direction of the current affects our result. We do that by adding the sine of the angle between the current and the radius. That way, if the wire is curvy, we'll take that into account. And that's it - that's the Biot-Savart law.

The magnetic field dB due to this element is to be determined at a point P which is at a distance r from it. Let θ be the angle between dl and the displacement vector r.

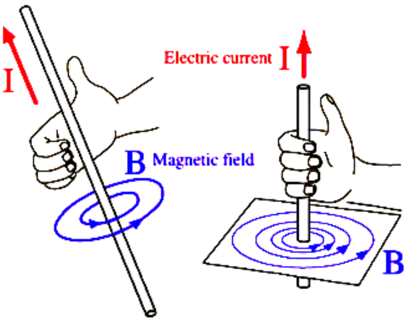
According to Biot-Savart's law, **the magnitude of the magnetic field dB is proportional to the current I, the element length |dl|, and inversely proportional to the square of the distance r**.

Its direction is perpendicular to the plane containing dl and r .

Thus, in vector notation:

$$dB \propto \frac{Idl \times r}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \times r}{r^3}$$

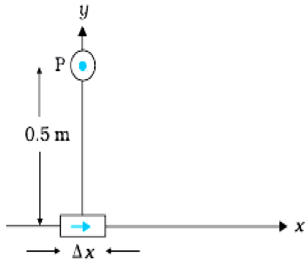


Direction of the field is given by **Right hand grip rule**

If you point your thumb in the direction of the current for this wire, your fingers will curl in the direction of the magnetic field. They'll follow the arrows of the concentric circles. And that's how you figure out the direction.

Application of Biot-savart law

- **Magnetic field due to a straight current carrying conductor a finite size**



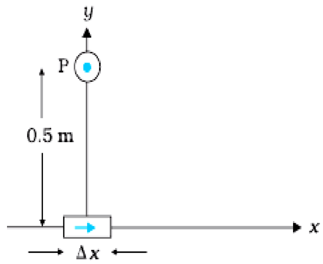
An element _l = *x* ˆ**i** is placed at the origin and carries a large current I = *10 A*. What is the magnetic field on the *y*−axis at a distance of 0.5 m. Δ*x* = 1 cm.

$$|dB| = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

$dl = \Delta x = 10^{-2}$ m , I = 10 A, r = 0.5 m = y, $\frac{\mu_0}{4\pi} = 10^{-7}Tm/A$

$\theta = 90^0,\ sin\theta = 1$

$$|dB| = \frac{10^{-7}\times10\times10^{-2}}{25\times10^{-2}} = 4\times10^{-8}T$$



The direction of the field is in the + z direction:

$dl \times r = \Delta x\hat{i} \times y\hat{j} = y\Delta x(\hat{i} \times \hat{j}) = y\Delta x\hat{k}$

This is because of the cyclic property of cross products:

$\hat{i} \times \hat{j} = \hat{k};\ \hat{j} \times \hat{k} = \hat{i};\ \hat{k} \times \hat{i} = \hat{j}$

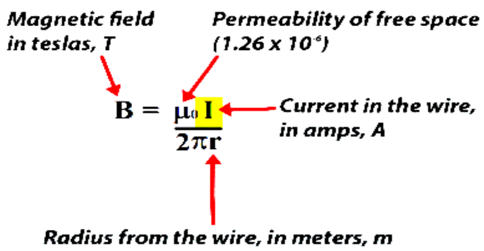
In effect for a straight wire we can say

$B = \mu_0 I / 2\pi r$

Or

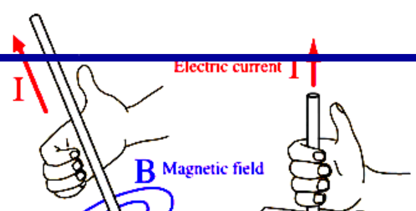
In other words, the magnetic field *B*, measured in tesla is equal to the permeability of free space μ_0, multiplied by the current going through the wire *I*, measured in amps, divided by 2π times the distance away from the wire *r*, measured in meters.

So this equation helps us figure out the magnetic field in a radius *r* from a straight wire carrying a current *I*. Notice the circle of radius r is perpendicular to the wire.



The equation gives us the magnitude of the magnetic field and direction by right hand grip rule.

This rule is used to know the direction of magnetic field due to a current-carrying conductor According to this rule, **"if we grasp a section of the wire conductor in our right hand such that the thumb points in the direction of current, then the fingers will encircle the conductor in the direction of the magnetic field."**



Example

A wire carrying a current of 0.1 amps, Calculate the magnetic field at a distance of 0.5 meters from the wire.

Solution

$I = 0.1$ A,

$r = 0.5$ m.

$\theta = 90^0$

so $B = ?$.

$B = \mu_0 I / 2\pi r$

$B = 10^{-7} \times .1 / .5 = 4 \times 10^{-8}$ Tesla

Think About These

- Would the magnitude of B depend upon the length of the conductor?
- Would the magnitude of B depend upon the orientation of the conductor?
- Would the magnitude of B depend upon the current through the conductor?
- Would the magnitude of B depend upon the environment around the conductor?
- Would the magnitude of B depend upon variation in the magnitude of current in the conductor?

- **Magnetic field on the axis of a circular current loop**

In this section, we shall evaluate the magnetic field due to a circular coil along its axis. The evaluation entails summing up the effect of infinitesimal current elements ($I\,dl$) mentioned in the previous section.

We assume that the current I is steady and that the evaluation is carried out in free space (i.e., vacuum).

Figure depicts a circular loop carrying a steady current I. The loop is placed in the y-z plane with its centre at the origin O and has a radius R.

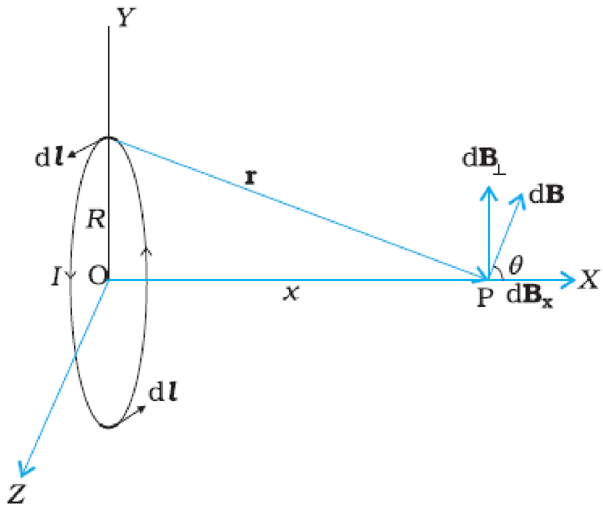
The x-axis is the axis of the loop.

We wish to calculate the magnetic field at the point P on this axis.

Let x be the distance of P from the centre O of the loop.

Consider a conducting element dl of the loop. This is shown in Figure. The magnitude dB of the magnetic field due to dl is given by the Biot-Savart law:

$$dB = \frac{\mu_0}{4\pi}\frac{I|dl\times r|}{r^3}$$



Notice

- $r^2 = x^2 + R^2$
- any element of the loop will be perpendicular to the displacement vector 'r' from
  - The element dl to the axial point.
  - The element dl in Figure is in the y-z plane whereas the displacement vector r from dl to the axial point P is in the x-y plane.

    Hence $|dl \times r| = r\, dl$. Thus,

    $$dB = \frac{\mu_0}{4\pi}\frac{Idl}{(x^2+R^2)}$$
- The direction of dB is shown in Figure. It is perpendicular to the plane formed by dl and r. It has an x-component dB_x and a component perpendicular to x−axis, dB⊥. (in the y−z plane)
- When the components perpendicular to the x-axis are summed over, they cancel out and the net value =0.
  - For example, the dB⊥ component due to dl is cancelled by the contribution due to the diametrically opposite dl element, shown in Figure
- The net contribution is along the x direction
  - Its magnitude is obtained by integrating $dB_x = dB\cos\theta$ over the loop.

$$\cos\theta = \frac{R}{(x^2+R^2)^{1/2}}$$

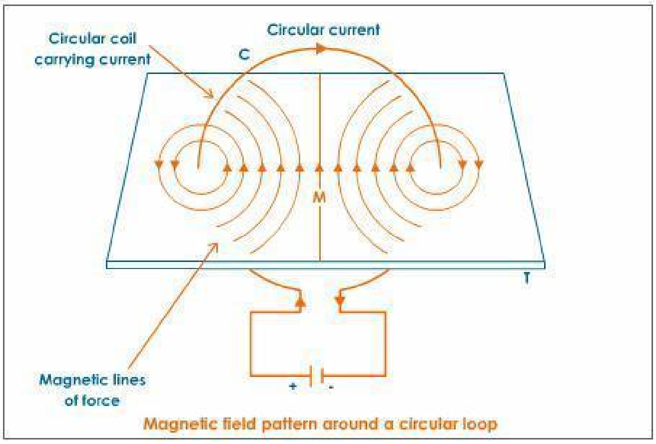
$$dB = \frac{\mu_0 Idl}{4\pi}\frac{R}{(x^2+R^2)^{3/2}}$$

The summation of elements dl over the loop yields $2\pi R$, the circumference of the loop. Thus, the magnetic field at P due to entire circular loop is:

$$B = B_x \hat{i} = \frac{\mu_0 I}{2} \frac{R^2}{(x^2+R^2)^{3/2}} \hat{i}$$

Special Cases:

- When point P is at the centre of the coil, then $B_0 = \frac{\mu_0 I}{2R} \hat{i}$



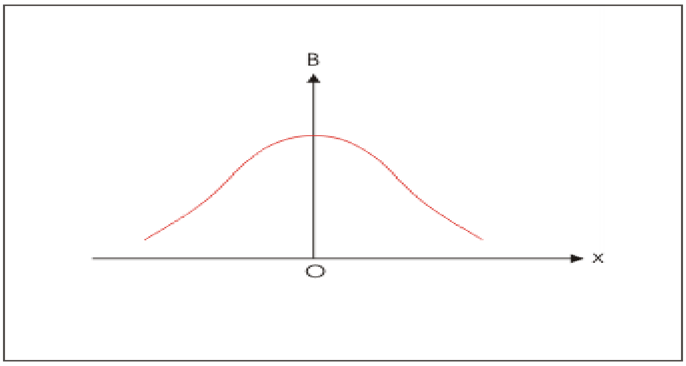
- When P is very far away from the centre of the coil, then x >>> R, so:

$$B = \frac{\mu_0 I R^2}{2x^3}$$

Here, the magnetic field is along the axis of the coil.

Variation of the magnetic field along the axis of a circular current loop.

The figure shows the variation of the magnetic field along the axis of a circular loop with distance from the centre. The value of B is maximum at the centre and it decreases as we go



away from the centre on either side of the loop. Variation of B along the axis of a circular current loop

Examples based on the application of Biot Savart law

Example

Consider a tightly wound 100 turns coil of radius 10 cm, carrying a current of 1A. What is the magnitude of the magnetic field at the centre of the coil?

Solution

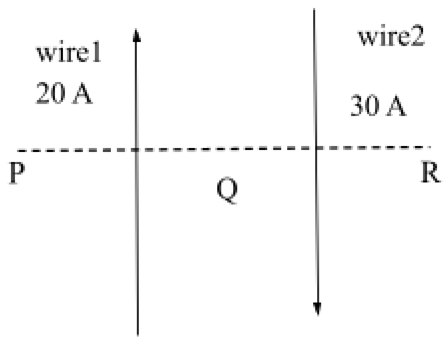
Since the coil is tightly wound, we may take each circular element to have the same radius R = 10 cm = 0.1 m.

The number of turns N = 100. The magnitude of the magnetic field at the centre is:

$B = \mu_0 NI / 2R = 4\pi \times 10^{-7} \times 10^{2} \times 1 / 2\times10^{-1} = 2\pi \times 10^{-4} = 6.28 \times 10^{-4}$ T

Example

In the figure are shown two current carrying wires 1 & 2. Find the magnitudes and directions of the magnetic field at the points at P, Q and R. The distance between p and wire 1 is 10 cm, between wire 1 and 2 is 20 cm and each point is 20 cm



Solution

The magnetic field at a point distant R metre from a long-straight wire carrying a current i ampere is given by

$B = \mu_0 NI / 2R = 2\times 10^{-7}$ i/R $NA^{-1}m^{-1}$

According to right hand rule the field at P due to the current in wire 1 will be perpendicular to the page pointing upward and that at Q & R pointing downward. Similarly, the field due to the current in wire 2 will be downward at P and Q, and upward at R. Thus at P and R the direction of magnetic fields due to the two wires are opposite but at Q they are in the same direction.

Therefore, **resultant field at P** is $B_P = B_1 - B_2$

$B_P = 2 \times 10^{-7}$ [20/0.1 – 30/0.3] $= 2\times10^{-5} NA^{-1}m^{-1}$

It will be perpendicular to the plane of the page pointing upward.

Resultant field at Q is $B = B_1 + B_2$

$B = 2\times10^{-7}[20/0.1 + 30/0.3]$

$B = 1\times10^{-4}\ NA^{-1}m^{-1}$

It will be perpendicular to the plane of the page pointing downward.

Resultant field at R is $B_R = B_2 - B_1$

$B = 2\times10^{-7}[30/0.1 - 20/0.3]$

$B = 4.7\times10^{-5}\ NA^{-1}m^{-1}$

It will be perpendicular to the plane of the page pointing upward.

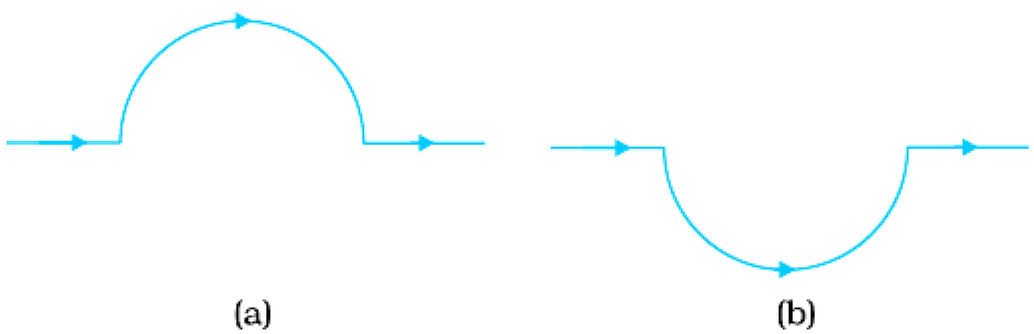
Think About These

1. What if the current in the two wires was equal?
2. What if it was in the same direction?

Example

A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown in Figure (a) Consider the magnetic field B at the centre of the arc.

a. What is the magnetic field due to the straight segments?
b. In what way the contribution to B from the semicircle differs from that of a circular loop and in what way does it resemble?
c. Would your answer be different if the wire were bent into a semi-circular arc of the same radius but in the opposite way as shown in Figure (b)?



Solution

a. dl and r for each element of the straight segments are parallel. Therefore, $dl \times r = 0$. Straight segments do not contribute to|B|.
b. For all segments of the semicircular arc, $dl \times r$ are all parallel to each other (into the plane of the paper). All such contributions add up in magnitude. Hence direction of B for a semicircular arc is given by the right-hand rule and magnitude is half that of a

circular loop.

Thus B is 1.9×10^{-4} T normal to the plane of the paper going into it.

c. Same magnitude of B but opposite in direction to that in (b).

Problem for Practice

1. In an orbital model of a hydrogen atom with one electron, revolving in circular orbit of radius 5.11×0^{-11} m at a frequency of 6.8×10^{15} Hz, what is the magnetic field at the centre of the orbit? Would the nucleus be subjected to this magnetic field?
   (Hint: current equivalent of the revolving electron with frequency n = I = ne)
2. The magnetic field due to a current carrying circular loop of radius 12 cm at its center is $0.50.\times 10^{-4}$ T .Find the magnetic field due to this loop at a point on the axis at a distance of 5.0 cm from the center.
   (Hint: find the ratio of field at the centre of the loop and field at a point on the axis of the loop).
3. Two coaxial circular loops of radii 2 cm and 4 am are laced such that their centres are 4 cm and 3 cm from a point O on the common axial line. The current in loop 1 is 1 A and is anticlockwise. What should be the magnitude and direction of current in loop 2 such that the net magnitude of B at O is zero**?**

Summary

We have learnt:

- Meaning of Biot-Savart' law.
- Applications of Biot Savart's law i.e., magnetic field due to straight conductor and special case to finite length.
- The magnetic field on the axis of the circular coil carries current followed by its special case to the magnetic field at its centre.
- Problem solving using Biot Savart's Law.